Measuring the World around us

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1 | Summary

Measuring the world around us implies being able to understand it. Answering the questions of a curious mind, this teaching unit shows how to measure distances and towering heights by means of simple reasoning. How often do we ask ourselves, how tall is that skyscraper, that mountain or that TV tower? Why not use the ubiquitous smartphone to find out? In this unit the students will make their measurements by using the appropriate apps, and they will compare their results with the measurement data gathered by using classic, well-established tools.

- **Keywords:** distance, height, altitude, parallax, pressure
- **Disciplines:** physics, mathematics, ICT
- **Age level of students:** 12-19 years
- **Android apps:** Distance and Parallax, Smart Distance, Distance and Height, Height and Pressure
- **iOS apps:** Easy Measure, Height and Distance, My Altitude

2 | Conceptual introduction

At the beginning of physics classes, the students are taught to measure lengths and distances. The well-known method of measuring distance or height is to use a ruler or measuring tape. In this teaching unit we propose that the students use other modern tools: various applications and a smartphone. They can decide on their own which way to go: to measure in the classic way, to use pen and paper to do the calculation or, alternatively, to use applications and a smartphone. The students will compare these methods and discover the advantages of each one.

The students can use different kinds of applications to make their measurements, but in this unit we have carefully chosen some examples for our purpose. These applications require some knowledge of mathematics (especially geometry) and physics formulas.

First, the students will become acquainted with the principle of parallax and then employ the Distance and Parallax Android application (as far as we know there is no corresponding iOS application) to determine the distance to a remote object.

Secondly, the students will determine the distance to the target object and also its height by using a ruler and the Distance and Height Android application or the Height and Distance iOS application.

Both activities will require the students to revise their basic knowledge of geometry, specifically the theorems about similar triangles, such as Thales' theorem.

Thirdly, the height of a hill or a mountain can be derived by measuring the temperature and the air pressure both at the base and at the top. In this case the students will use the Height and Pressure Android application, which was created for this teaching unit, or the My Altitude iOS application.

These smartphone apps enable the students to use various methods of investigation to reach their goals, such as mathematical computation, physics formulas, comparisons and checks with data that is available online, in order to find various geographic altitudes or distances.

This unit fits in well with European curricula.

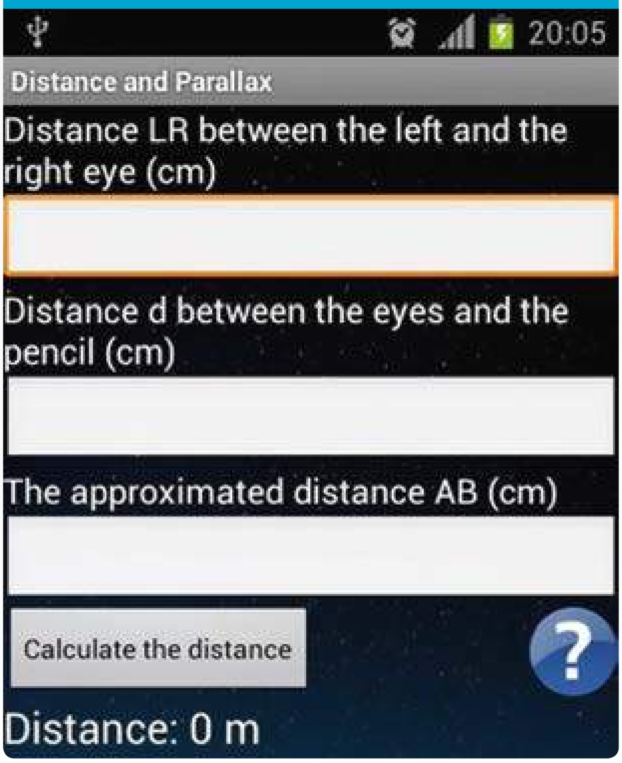
3 | What the students do

3 | 1 How to use the Android application Distance and Parallax in order to find the terrestrial distance to a remote object

The students can find a complete explanation of the parallax effect and the use of the parallax method in terrestrial measurements in the teaching unit "Smartastronomers: From the Classroom to the Sky". The new method is described in paragraph 3.1 (page 9).

The observer must first measure the distance between his eyes and his thumb d (or pencil) and the distance between his eyes LR. With the help of these values, the observer can

FIG.1 Screenshot: Distance and Parallax



then estimate the distance AB (the approximate distance between the perceived locations of the targeted object). The students have to insert the data into the respective fields of the application (**FIG. 1**) and then push the button **<CALCULATE THE DISTANCE>** to obtain the distance.

In this case, with the following data: d = 55 cm, AB = 3 m and LR = 6.5 cm, the result for the length of the school's sport field was D = 25.9 m.

To be sure that the measurement above was good, the students measured the same distance with a classic method using a measuring tape. They found that D = 25 m.

The students can compare the results and decide which measurement is easier to do.

The students can use various other applications, such as Smart Distance for Android and Easy Measure for iOS, to determine the distance to an object. Such applications become essential when students cannot use a measuring tape to determine the direct distance to the target object (when the distances are too long). With this telemeter application, the measurement is made by using the smartphone's camera perspective. The students have to estimate the height of the target, enter this data into the respective field and then touch the screen and align the target between two horizontal lines. The smartphone will provide the distance value.

3 | 2 How to use the Distance and Height app for Android or the Height and Distance app for iOS to simultaneously determine the distance to the target object and its height

The students start by using a ruler to measure the apparent height h of the target object. The target object is situated at an unknown distance D.

They move several steps toward the target object. The distance d can be measured by using a measuring tape or estimated by using the approximate length of one's stride.

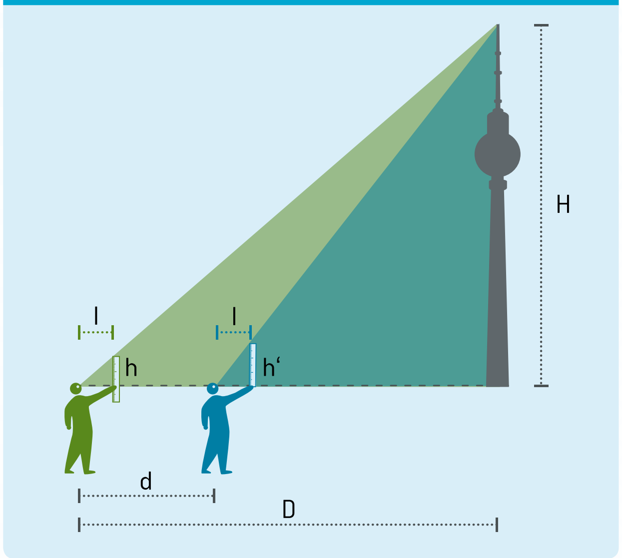
Once again, the students measure the apparent height of the target object h'. This time the measured apparent height will appear larger than the initial measurement.

The two trinangles are both right-angled triangles.

Distance D can be calculated by using the formula

$$D = d\,\frac{h'}{h'-h}$$

FIG. 2 Using a ruler to measure the apparent height



The target object height H can be calculated by using the formula

$$H = \frac{d \cdot h \cdot h'}{l(h'-h)}$$

where

- l – distance from the eyes to the ruler
- D – distance to the target object
- H – height of the target object
- d – distance between the two observation points
- h and h' – apparent heights of the target object, measured with a ruler.

Obviously it is much easier to input the data into a ready-made application that will instantly return the results (**FIG. 3**).

The students will observe that if the target is at a distance of about 50 m they can use a distance d of about 10 m between the two observation points. If the target is far away, about 2 km, then the distance d has to grow to 50 m and so on. The difference between the two apparent heights h and h' (found with the ruler) has to be at least 0.5 cm, because the human eye cannot accurately distinguish distances below this threshold.

FIG. 3 Screenshot: Distance and Height

3 | 3 How to use the Height and Pressure application for Android or My Altitude for iOS to determine the height of a mountain or the altitude of a school with a smartphone that incorporates temperature and barometric sensors

First of all the students have to see whether their smartphones are suitable. They can find out how many sensors their smartphones have by using the Sensor Box application for Android or the Sensor Monitor for IOS.

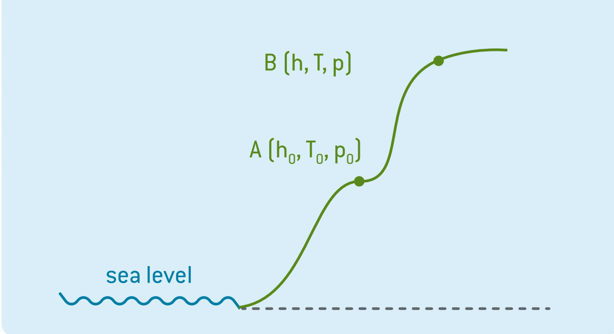
3 | 3 | 1 Height measurement with four parameters

The temperature and the pressure of the atmosphere vary with the altitude. If the students are measuring the atmospheric parameters at different altitudes, namely points A and B (**FIG. 4**), the temperature and the altitude can be considered to have a linear dependence within the troposphere.

At each kilometre from the sea level, there will be a temperature drop of 6.5 Kelvin. In this case the following equation can be used:

$$T = T_0 + \alpha(h - h_0)$$

FIG. 4 Measurements of atmospheric parameters at different altitudes



where

- T – temperature at altitude h (point B)
- T_0 – starting temperature at the starting altitude h_0 (point A)
- α – temperature gradient = –0.0065 K/m.

The relationship between pressure and temperature is expressed as follows:

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{-\frac{g\mu}{\alpha R}}$$

where

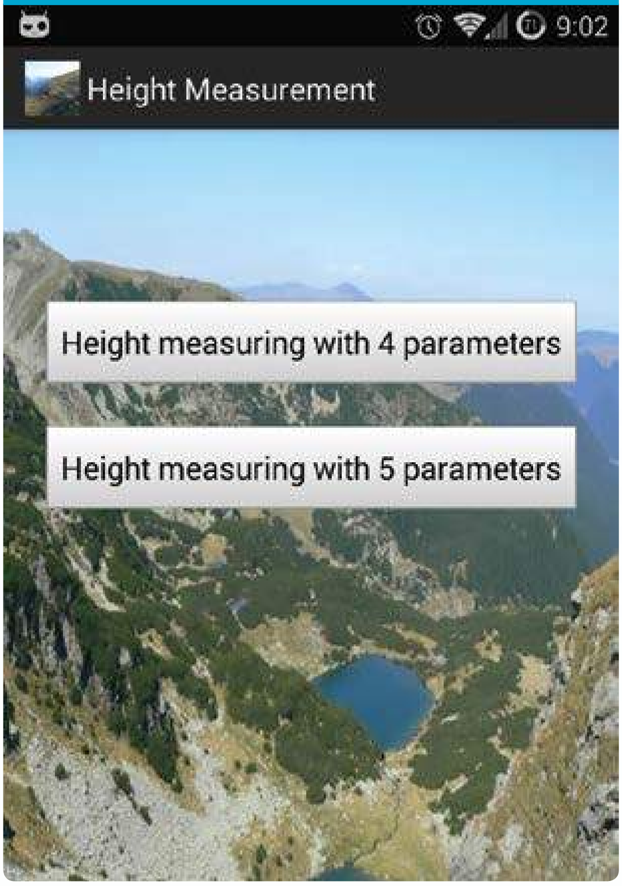
- p – pressure (in mbar) at altitude h
- p_0 – pressure (in mbar) at altitude h_0
- g – gravitational acceleration at sea level = 9.81 m/s2
- R – universal gas constant = 8.310 J/(kmol · K)
- μ – molar mass for the air = 29 kg/kmol.

By combining these two formulas, the students will obtain the final equation, which they can use to calculate the altitude:

$$h = \frac{T_0}{\alpha}\left[\left(\frac{p}{p_0}\right)^{-\frac{\alpha R}{\mu g}} - 1\right] + h_0$$

To recap: The students have to use their smartphones to measure the temperature T_0 and the pressure p_0 at the starting point A, find the altitude h_0 on the Internet (or to consider $h_0 = 0$) and then measure the pressure p at the final point B. They have to insert all of these data into the respective fields of the application and then push the button **<CALCULATE THE HEIGHT>**.

FIG. 5 Screenshot: Height Measurement



3 | 3 | 2 Measuring height with five parameters

By using the same application, the students can also calculate the height or the altitude using the barometric **formula 6**

$$p = p_{sl} \cdot e^{-\mu g h/(RT)}$$

where p_{sl} = pressure at sea level.

We obtain the following results for the parameters of both points A and B:

$$\frac{p}{p_0} = e^{-\frac{\mu g}{R}\left(\frac{h_0}{T_0} - \frac{h}{T}\right)}$$

In this case the students can calculate the altitude h by means of the following **formula 8**:

$$h = \frac{RT}{\mu g} \ln \frac{p_0}{p} + \frac{T}{T_0} h_0$$

Why use two formulas for the same measurement?

The students have to observe the similarities and the differences between these two methods.

First of all, if they use the second part of the application, they have to insert one more parameter: the temperature T at the point B.

Then they can complete a table (FIG. 6) with the data from the Internet and the data calculated with the Height and Pressure application.

Further experiments

It is advised that students use two or three different smartphone apps for measuring:

- Distances to the target objects
- Heights of target buildings
- Distances to geographic features (hills or mountains) and their heights

During their investigations they will notice that some applications are better suited for objects that are closer to the observer, while others are better for more distant objects.

They will be able to compare their results with reference data and then to identify the errors in their measurements.

The students could form teams to answer a number of simple questions:

- Which are the best applications for measuring small distances or large distances?
- Which application is best suited for measuring the height of the school building?

FIG. 6 Table of comparision of data

T_0 (K)	T (K)	p (mbar)	h_0 (m)	$h_{(formula\ 6)}$ (m)	$h_{(formula\ 8)}$ (m)	$h_{internet}$ (m)

Each student measures the height of the building or the height from a window of the building by using a piece of string that has a weight tied to one end and then measuring the length of the string. The string is hung from the top of the building or out of the window to its base. Afterwards the same height will be determined by using the apps presented in this teaching unit. In addition, the students can use a third application such as Smart Distance.

For the applications that employ geometric theorems, it will be useful to note that the final result will depend on the distance between the observer and the target object.

When measuring the height or altitude by using the pressure sensor, it is interesting to compare the pressure measurements with the values measured by a classic barometer.

- Which application is suitable for determining your height?
- Which application is suitable for determining the height of a mountain?

3 | 4 Another interesting problem/exercise

Today almost every smartphone has a built-in accelerometer sensor. The students could potentially measure the gravitational acceleration on the ground and during an airplane flight. By using the formula for gravitational acceleration, which depends on altitude, the students can derive the altitude of the plane. Can this method be employed successfully? Provide arguments for and against its successful use.

4 | Cooperation option

Students from different countries can compare their results and initiate a common project with a title such as “Measuring the height of your school with a smartphone”.

5 | Conclusion

Even though this teaching unit may appear relatively easy at first glance, the investigations require the students to refresh their knowledge of basic geometry, mechanics and hydrostatics, while recording and processing a fair amount of data.

The students have to know the mathematical formulas and the physical laws that form the foundation of the smartphone apps.

It is important for the students to choose the right smartphone application to use for a given range of heights or distances so that they can achieve the most accurate measurements. They will realise that the smartphone is an invaluable tool for this purpose.

Last but not least, we would like to emphasise that the three Android applications used in this teaching unit (Distance and Parallax, Distance and Height, and Height and Pressure) were developed by Alex Toma, a Romanian student. It’s a challenge for students to try to develop corresponding iOS applications or others that could be used in experiments. This means that physics, mathematics and computer science would have to work together to develop interesting new methods for studying the natural sciences.